# The Milky Way, The Local Group & the IR Tully-Fisher Diagram

Sangeeta Malhotra<sup>1,3</sup>, David N Spergel<sup>2,4</sup>, James E Rhoads<sup>2</sup>, Jing Li<sup>1</sup>

### ABSTRACT

Using the near infrared fluxes of local group galaxies derived from Cosmic Background Explorer (COBE)/Diffuse Infrared Background Experiment (DIRBE)<sup>1</sup> J(1.25  $\mu$ m) K (2.2  $\mu$ m) & L (3.5  $\mu$ m) band maps and published Cepheid distances, we construct Tully-Fisher diagrams for the Local Group. The measured dispersions in these luminosity-linewidth diagrams are remarkably small:  $\sigma_J = 0.09$  magnitudes,  $\sigma_K = 0.13$  magnitudes, and  $\sigma_L = 0.20$  magnitudes. These dispersions include contributions from both the intrinsic Tully-Fisher relation scatter and the errors in estimated galaxy distances, fluxes, inclination angles, extinction corrections, and circular speeds. For the J and K bands, Monte Carlo simulations give a 95% confidence interval upper limit on the true scatter in the Tully-Fisher diagram of  $\sigma_J \leq 0.35$  and  $\sigma_K \leq 0.45$ .

We determine the Milky Way's luminosity and place it in the Tully-Fisher diagram by fitting a bar plus exponential disk model of the Milky Way to the all-sky DIRBE maps. For "standard" values of its size and circular speed (Sun-Galactic center distance  $R_0 = 8.5 \mathrm{kpc}$  and  $\Theta_0 = 220 \mathrm{km/s}$ ), the Milky Way lies within  $1.5\sigma$  of the TF relations. We can use the TF relation and the Cepheid distances to Local Group galaxies to constrain  $R_0$  and  $\Theta_0$ :

-  $\log (R_0/8.5 \mathrm{kpc}) + 1.63 \log (\Theta_0/220 \mathrm{km/s}) = 0.08 \pm 0.03$ . Alternatively, we can fix the parameters of the Galaxy to their standard values, ignore the Cepheid zero point, and use the Tully-Fisher relation to determine the Hubble Constant directly:  $H_0 = 66 \pm 12 \mathrm{~km/s/Mpc}$ .

We have also tested the Tully-Fisher relation at longer wavelengths, where the emission is dominated by dust. We find no evidence for a Tully Fisher relation at wavelengths beyond  $10\mu\mathrm{m}$ . The tight correlation seen in L band suggests that stellar emission dominates over the 3.3  $\mu\mathrm{m}$  PAH emission.

Subject 11 cadings: Gal axy: fundamental parameters Galaxy: general galaxies: distances and redshifts

<sup>&</sup>lt;sup>1</sup>COBE data sets were developed by the NASA Goddard Space Flight Center under the guidance of the COBE Science Working Group and were provided by the NSSDC.

## 1. Introduction

The Tully-Fisher (TF) relation between the luminosity and linewidth of spiral galaxies (Tully & Fisher 1977) has been used extensively as a distance indicator and to map large scale flows of galaxies (cf., Strauss & Willick 1995 and Jacoby et al. 1992 for reviews). Its usefulness as a distance indicator is limited by the intrinsic scatter of the relation. This scatter is lowest in redder bands where dust extinction is low: Il band (1.65µm) (Aaronson, Huchra & Mould 1979, Aaronson, Mould & Huchra 1980, Aaronson et al. 1989, Freedman 1990, Pierce & Tully 1992) and I band (0.90µm) (Bernstein et al. 1994). In this paper, we extend the TF relation to longer wavelengths. The DIRBE experiment, with its excellent calibration and large beam width, is ideal for measuring the total flux of galaxies in the Local Group. We describe our Local Group data set and present the results of our analysis in section 2.

The Milky Way has often been deemed unsuitable for zero point calibration of the TF relationship, mainly because of difficulties in estimating its total luminosity. Such difficulties can be overcome at infrared wavelengths, where dust absorption is small. In section 3, we use a three-dimensional model of the Milky Way based on the DIRBE J, K, and L band maps to obtain a measurement of the Galaxy's luminosity. This luminosity can be used to place the Milky Way on the Local Group TF diagram, to constrain Galactic parameters, and to obtain an independent calibration of the Hubble constant.

# 2. The Local Group

For this study, we chose a sample of nearby bright spiral galaxies with sizes not too n smaller than the beam, no easured Cepheid distances, no bright stars in a 1° field centered on the galaxy, and with inclination greater than 45\*. We also required that the measured DIRBE flux from the galaxy exceed the week-to week fluctuations in the DIRBE data. The galaxies used are M31, M33, M81, NGC 300, and NGC 2403. A sixth galaxy, NGC 247 shows a marginal detection and meets our selection criterion in only 2 bands (J and L). We will do our analysis both with and without NGC 247. Barring M31 and M33, these galaxies are smaller than the 0.7° x 0.7° DIRBE beam. M33 is slightly larger than the

<sup>&</sup>lt;sup>1</sup>IPAC, 100-22, California Institute of Technology, Pasadena, CA 91125

<sup>&</sup>lt;sup>2</sup>Princeton University Observatory, Princeton NJ 08540

<sup>&</sup>lt;sup>3</sup>N RC Fellow at NASA Jet Propulsion Laboratory

<sup>&</sup>lt;sup>4</sup>Department of Astronomy, University of Maryl and, College Park, MD 20742

beam, and M31 and Milky Way are extended objects compared to the beam.

The measured fluxes were corrected for extinction using the prescription of Tully & Fouqué (1985), applied to our sample using inclination angles from Pierce & Tully (1992) and the standard extinction law from Mathis (1990). These corrections were never large, attaining maximum values of 0.18, 0.07, and 0.03 magnitudes in J, K, and L for M31. The distances and distance errors were taken from published literature as follows: M31 (Freedman 1990), M33 (Freedman, Wilson & Madore 1991), M81 (Freedman et al. 1994, Hughes et al. 1994), NGC247 (Catanzarite, Freedman & Madore 1996), NGC300 (Freedman 1990) and NGC2403 (Freedman 1990). Line widths for these galaxies are taken from Pierce & Tully (1992) and Freedman (1990).

# 2.1. Flux Extraction

DIRBE was one of the three experiments on board the COBE mission. This experiment observed the entire sky in ten wavebands (from 1.25  $\mu$ m to 240  $\mu$ m) over a nine month period. Each point on the sky was observed many times, at a range of solar elongation angles. This gives a variable viewing geometry from within the interplanetary dust cloud, which is a major source of foreground emission at these wavelengths. A detailed description of the COBE mission can be found in Boggess et al. (1992). Details of DIRBE data processing, calibrations and photometry are given in the DIRBE Explanatory Supplement (Hauser et al. 1995).

The fluxes of the galaxies were derived from the weekly sky maps provided by the DIRBE team. Intensity in each pixel of the weekly map, which is typically half a beam across, is a robust average of all observations in a week pointing at the region of the sky in that pixel. Thus the center of the beam for any observation may be up to half a pixel away from the center of the pixel, and does not in general lie on the center of the galaxy which lies in that pixel. Since the beam shape is not flat this can influence the flux measured for that galaxy. This error is estimated to be a few to nine percent in the DIRBE Explanatory Supplement (section 5.6.6).

For each weekly map we fit a first order polynomial to the sky brightness in a small annulus around the pixel that contains the galaxy. The sky background was then subtracted from the maps. A point source can influence the flux levels of neighboring pixels due to the way the weekly maps were made, so pixels adjacent to galaxies were excluded from the sky estimate. All the weekly maps which had observations of that part of the sky were then averaged (after rejecting outliers) to give the the average flux of the galaxy. The week to

week variation of the flux in a given pixel gives an estimate of the noise in the maps. The flux per pixel thus obtained in MJy/Sr is multiplied by the beam size (given in the DIRBE Explanatory Supplement) to derive the flux of the sources.

The flux in a given pixel has contributions from the galaxy, unresolved Galactic stars and zodiacal light. The zodiacal light varies from week to week as one sees the same part of the sky through a different path length of the zodiacal dust cloud near the Earth. This makes it necessary to remove the background separately for each week. We assume that the starlight is smoothly varying in space and can be subtracted as part of the background. This is true except in the rare case of a very bright star close to the galaxy. We examined the Palomar Observatory Sky Survey plates for such bright stars near candidate galaxies, and rejected NGC 925 and NGC 4571 from our sample because of nearby bright stars. The week to week variation in the flux of the source gives the estimate of errors due to imperfect zodiacal light subtraction, and also due to the beam being centered at different positions with respect to the galaxy. We have used the week-to week flux variation as a conservative estimate of the errors in the flux measurement.

M33 is slightly bigger than the beam size, and the single pixel method described above underestimates its flux. We simulated observations of M33 assuming an exponential disk with a scale length of 6' (Regan & Vogel 1994) and the beam profile of the DIRBE beam. In these simulations, we placed the beam randomly within the pixel closest to the position of M33 center. The same experiment was repeated with a point source. The flux measured for the M33 simulation was compared to the flux measured for a point source and a correction factor of -0.55 magnitude was added to the M33 magnitude. M31 appears extended in the DIRBE maps, so its flux was measured by summing all the pixels containing the galaxy and normalizing this sum by the ratio of the pixel area to the beam area.

# 2.2. MaximumLikelihood Analysis

We determined the TF parameters and errors using a maximum likelihood analysis. This approach is preferable to a  $\chi^2$  fit to the data as the maximum likelihood analysis includes the selection bias due to our magnitude limit (Willick 1994). In a magnitude limited sample, selection bias implies that galaxies near the luminosity cut-off are systematically brighter (Willick 1994). N GC 247, which falls just below our cut-off, behaves just as expected given this effect: it is brighter than the best fit TF relation. We perform our analysis both with and without NGC 247.

The maximum likelihood function for our sample is,

$$P(\tilde{m}|a,b,\sigma_{TF}) = \prod_{i=1}^{n} \frac{\exp\left\{-\frac{[m_i,D_i,[a_{m_i}+b]]^2}{2(\sigma_{TF}^2)}\right\}}{\int_{-\infty}^{m_{tf}} \exp\left\{-\frac{[m_i,D_i,[a_{m_i}+b]]^2}{2(\sigma_{TF}^2)}\right\} dm'$$

parameters a, b, and  $\sigma_{TF}$  are the slope, zero point, and [total] scatter about the Tully-Fisher amplitude of the week-to-week fluctuations measured at the galaxy location). The model moduli  $D_i$ , the logarithms of their line widths  $\eta_i$ , and the magnitude limits  $m_{l,i}$  (i.e., the corrections, and circular speeds. The product is evaluated over all galaxies in the sample, scatter and the errors in estimated galaxy distances, fluxes, inclination angles, extinction relation. Note that  $\sigma_{TF}$  includes contributions from both the intrinsic Tully-Fisher relation Here the observed quantities are the galaxies' magnitudes  $m_i$ , Cepheid-based distance .e., all galaxies for which  $m_i < m_{l,i}$ .

The maximum likelihood fits to the J, K and L band Tully Fisher relations (Figures

$$J^c:=8.13^{+0.41}_{-0.42}\log(\Delta V/400\text{km/s})-22.00^{+0.12}_{-0.12}$$
  
 $K^c:=8.59^{+0.67}_{-0.65}\log(\Delta V/400\text{km/s})-23.01^{+0.11}_{-0.11}$   
 $L^c:=9.01^{+0.96}_{-0.94}\log(\Delta V/400\text{km/s})-22.99^{+0.14}_{-0.17}$ 

minimum. A small scatter in the TP relation of 5 galaxies implies a small intrinsic scatter in J band,  $\sigma_{TF}(K) = 0.13$  magnitudes in K band and  $\sigma_{TF}(L) = 0.20$  magnitudes in L in the TF relation for all galaxies. We confirm this for our sample with a Monte Carlo program. Investigation of the likelihood surface showed that it was well-behaved near its likelihood function was done with the Numerical Recipes (Press et al. 1992) AMOEBA Fisher, relation in these bands is very tight, showing a scatter of  $\sigma_{TF}(J) = 0.09$  magnitudes where  $J^c, K^c$  and  $L^c$  are dust-corrected magnitudes. The inewidth luminosity, or Tully simulations. The quoted errors on the TF slope and zero point are the simulations. For  $\sigma_{TF}(J) = 0.55$ , we recovered such a small scatter in only a true  $\sigma_{TF}(J) = 0.35$ , we recovered a scatter as small as observed in less than 5% of the reported  $\eta_i$ , and our best fit slopes. We varied the value of  $\sigma_{TF}$  in our simulations. For simulation with 1000 realizations of the Local Group. We generated 5 galaxies with the band.  $\chi^2$  fits yield smaller (but less believable) values for  $\sigma$ . The minimization of the from the Monte Carlo simulations

If we include NGC 247 in the sample, then the maximum likelihood fits are

$$J^c : -7.78 \log(\Delta V/400 \text{km/s}) - 22.07$$
,

$$K^{c} = -8.90 \log(\Delta V/400 \text{km/s}) - 22.99$$
,

$$L^c = -8.76 \log(\Delta V/400 \text{km/s}) - 23.00$$
,

and the corresponding Tully Fi sh er relation scatter is  $\sigma_J = 0.15$ ,  $\sigma_K = 0.14$ , and  $\sigma_L = ()$ . 2.1 magnitudes. For a true  $\sigma_{TF}(K) = 0.45$ , we found such small scatter in less than 5% of simulations. For  $\sigma_{TF}(K) = 0.7$ , we found such smaller scatter in only 1% of simulations.

The dust emission at longer wavelengths 60-240  $\mu m$  does not show a luminosity-linewidth relation. From this and from the tight L band Tully Fisher correlation, we conclude that the non-stellar or dust contribution to the L band is not large. This is consistent with recent estimates of 8-16 % contribution of the 3.3  $\mu m$  PAH feature to the L band (Bernard et al. 1994)

While giant stars are thought to contribute most of the light in near IR bands, supergiants can make an important contribution in the spiral arms (Rhoads 1996). This contribution does not appear to be a source of significant scatter in the TF relation.

# 3. The Milky Way

We cannot directly use the total flux from the Milky Way (MW) as regions at different distances contribute to that flux. We estimated the luminosity density of the MW from the best fitting model to the DIRBE data. The MW was modelled as a sum of an exponential disk and a bar (Spergel, Malhotra & Blitz 1996, Paper I). Extinction corrections were based on a 3 dimensional dust model. The uncertainty in the estimated brightness of the MW due to modelling is about 10% (Paper I). For Galactic parameters, we assume a circular speed  $\Theta_0 = 220 \pm 10$  km/s and a distance to the Galactic center  $R_0 = 8.5 \pm 0.5$  kpc (Gunn, Knapp & Tremaine 1979). Using these values, we obtain absolute magnitudes J = -23.05, K = -24.06, and L = -23.88. More recent determinations of the Sun-Galactic center distance using water masers suggest a smaller distance:  $7.1 \pm 1.5$  kpc (Reid et al. 1988) and  $8.1 \pm 1.1$  kpc (Gwinn, Moran & Reid 1992). Combining our uncertainties and assuming a TF slope of 9 implies an intrinsic uncertainty in the Galaxy's position in the TF diagram of 0.23 magnitudes.

If we include Milky Way in the sample, then the maximum likelihood fits are:

$$J^c = 8.09 \log(\Delta V/400 \text{km/s}) - 22.15$$
  
 $K^c = -9.23 \log(\Delta V/400 \text{km/s}) - 23.03$   
 $L^c = 8.91 - \log(\Delta V/400 \text{km/s}) - 23.04$ 

For uniformity, the velocity width of the MW was scaled from the observed velocity width of M31:  $\Delta V(MW) = (220/250) \times \Delta V(M31)$ . The Tully Fisher relation in these bands

remains very tight with  $\sigma_J = 0.18$ ,  $\sigma_K = 0.16$ , and  $\sigma_L = 0.21$ . We repeated the Monte Carlo simulations as described in Section 2. For a true  $\sigma_{TF}(K) = 0.45$ , we found a scatter as small as observed in less than 5% of simulations. For  $\sigma_{TF}(K) = 0.57$ , we found such a small scatter in only 1% of simulations. With the standard parameters, the Milky Way is  $\sim 1.5\sigma$  too luminous in J,  $\sim 1\sigma$  too luminous in K, and  $\sim 0.3\sigma$  too luminous in L. This basic agreement shows that the Cepheid distance scale is consistent at the 10% level with Gunn et al. (1979) distance determination to the Galactic center.

Alternatively, we can use the TF relation obtained for the 5 galaxy Local Group sample, together with our model for the Galactic luminosity to constrain Galactic parameters:

$$-5\log\left(\frac{R_0}{8.5\text{kpc}}\right) + 8.13\log\left(\frac{\Theta_0}{220\text{km/s}}\right) = 0.42 \pm 0.16$$

$$-5\log\left(\frac{R_0}{8.5\text{kpc}}\right) + 8.59\log\left(\frac{\Theta_0}{220\text{km/s}}\right) = 0.39 \pm 0.19$$

$$-5\log\left(\frac{R_0}{8.5\text{kpc}}\right) + 9.01\log\left(\frac{\Theta_0}{220\text{km/s}}\right) = 0.20 \pm 0.24$$

where the 3 equations correspond to fitting the J, K and L band relations respectively. The uncertainties include the width of the TF relation, the quoted uncertainties in the Galactic model (10%) and a conservative estimate of systematic errors in point source flux measurements (9%). This comparison of the Galaxy with the other Local Group galaxies suggests that its rotation speed is unlikely to be as small as 200 km/s.

Finally, we can also use the Galaxy (if we are willing to assume values for  $R_0$  and  $\Theta_0$ ) to obtain a value of the Hubble parameter that is independent of the zero point of the Cepheid distance scale (Wright 1 994). If we use the Galaxy as the zero point of the K band Tul ly Fisher relation, we find that

$$K^c = -8.59[\log(\Delta V) - 2.5] - 22.52 \pm 0.29 - 5\log\left(\frac{R_0}{8.5 \text{kpc}}\right) + 8.59\log\left(\frac{\Theta_0}{220 \text{km/s}}\right)$$

where the zero point error is the sum in quadrature of the uncertainties in Galactic parameters, the systematic error in the point source flux measurements, the uncertainties in the Galaxy model, and the deviation of a typical galaxy from the TF relation.

of external galaxies (A aronson et al. 1986), we use measured fluxes for large 11111111 ers of external galaxies (A aronson et al. 1986), we use measured 11 band fluxes for our sample (Freedman 1990, Pierce & Tully 1 9 90) to determine the dust-corrected  $H^c K^c$  and  $J^c - K^c$  color-color relation for galaxies in the Local Group. A  $\chi^2$  fit to the data yields  $(H^c - K^c) = 3.27 - 2.50 \times (J^c K^c)$  with a scatter of -0.18 magnitudes. This suggests

obtained by using Cepheid distances and the Local Group to normalize the TF relation with  $\log(\Delta v) = 2.5$ . If we use this estimate to set a zero point for the H band TF relation  $(H^c - K^c) = 0.75 \pm 0.18$  for the Milky Way and therefore,  $H^c = -21.77 \pm 0.34$  for a galaxy to the rest of the Local Group (for the standard Galactic parameters and the standard Table 6 and Equation 4), this yields  $H_0 = 66 \pm 12 \text{ km/s/Mpc}$ . This value is lower than that and combine it with the Aaronson et al. (1986) TF study of the local velocity field (their Coma cluster would yield a more definitive value of the Hubble constant. extrapolation from J and K band fluxes to uncertainty in Galactic parameters, the dominant source of error in these estimates is the  $\Theta_0/R_0$  constant, then the Hubble constant estimate decreases to 58 km/s/Mpc. Besides the to 79 km/s/Mpc. If we assume that  $R_0 = 7.1$  kpc and hold the angular rotation rate, [Reid et al. 1988]) and hold  $\Theta_0$  constant, then the Hubble constant estimate increases Cepheid distances). If we assume that  $R_0 = 7.1 \text{ kpc}$  (the smaller maser-based measurement (Aaronson et al. 1986, Freedman 1990) as the Galaxy is somewhat over-luminous relative band. Thus, a K band survey of spirals in

# 4. Conclusions

part of the sky and nearer to the plane of the Milky Way. distances and hence the peculiar velocity flows for many more galaxies (  $\sim 10^6$ ) in a greater two major near-IR sky surveys (2MASS and DENIS), there is also potential to estimate the usefully exploited for distance estimation. With the advent of imaging IR instruments and are about half and one-third as large as for the H-band at  $1.65\mu\mathrm{m}$ , so these bands may be previously explored. The extinction corrections (cf. Mathis 1990) at 2.2  $\mu \rm m$  and 3.5 $\mu \rm m$ We find that TP relation extends to longer wavelengths (2.2  $\mu$ m and 3.5  $\mu$ m) than

Group TF relation. This consistency is an independent check of the distance scale Way) and assuming standard Galactic parameters, we find that the Galaxy obeys the Local Using Cepheid distances for galaxies in the Local Group (not including the Milky

# 5. Acknowledgements

about the DIRBE data. We also thank Michael Strauss for helpful comments on an earlier Ned Wright for discussions and suggestions, and David Leisawitz for answering questions This wo k was partially supported by NASA A JP NAG5 269. version of this manuscript. Lais work made use of NASA Extragalactic Database N 5 D We thank Leo Blitz, George Helou, Jill Knapp, Barry Madore, Nancy Silbermann and

### REFERENCES

Aaronson, M., Huchra, J. & Mould, J. 1979, ApJ, 229, 1

Aaronson, M., Mould, J. & Huchra, J. 1980, ApJ, 237, 655

Aaronson, M. et al. 1986, ApJ, 302, 536

Bernard, J.] '., Boulanger, F., Désert, F. X., Giard, M., Helou, G. & Puget, J.L. 1994, AA, 291, L5

Bernstein, G.M, Guhathakurta, 1'., Raychaudh ary, s., Giovanelli, R., Haynes, M.P., Herter, T.& Vogt, N. 1994, AJ, 107, 1962.

Boggess, N. et al. 1992, ApJ 397, 420

Catanzarite, J.11., Freedman, W.I., & Madore, I.I., 1 996, in preparation

Freedman, W.L.1990, 355, L35.

Freedman, W.L, Wilson C.D. & Madore, B. 1991, ApJ, 372, 4.55

Freedman, W.], et al. 1994, ApJ, 427, 628.

Gwinn, C.R., Moran, J.M. & Reid, M.J. 1992, ApJ, 393, 149

Gunn, J.E., Knapp, G.R. & Tremaine, S. 1979, AJ, 84, 1181

Hauser, M.G., Kelsall, T., Leisawitz, D., Weiland, J. 1995. The DIRBE Explanatory Supplement, from "http://www.gsfc.nasa.gov/astro/cobe/dirbe.exsup.htmp"

Hughes, S. et al. 1994, ApJ, 4%, 143

Jacoby, G., Branch, D., Ciardullo, It., Davies, R., Harris, W., Pierce, M., Pritchet, C., Tonry, J. & Welch, D.1992, I'ASI', 104, 599.

Mathis, J.S. IWO, ARAA, 28, 37.

Pierce, M.J. & Tully, R.B., 1992, ApJ, 387, 47.

Press, W.H., Teukolsky, S. A., Vetterling, W.T., & Flannery, 13.P. 1 992, Numerical Recipes in Fortran: The A-rt of Scientific Computing.

Regan, M. & Vogel, S., 1994, ApJ, 434, X36.

Reid, M. J., et al. 1988, ApJ, 330, 809.

R hoads, J.E., 1996, submitted to the ApJ.

Spergel, D.N., Malhotra, S. & Blitz, 1,., 1996, in preparation

Strauss, M.A., & Willick, J. A., 1995, Physics Reports, 261, 271

Tully, J.B., Fouqué, J., 1985, ApJS, 58, 67

Tully, J.B, Fisher, J.R., 1977, AA, 54, 661Willick, J.A. 1994, ApJS, 92, 1.Wright, E. 1994, BAAS, 185, 32.05.

This preprint was prepared with the AAS IMPX macros v4.0  $\,$ 

Fig. 1. The J band (1.25 $\mu$ m) Tully-Fisher diagram. The error bars for the Local Group galaxies M31, M81, NGC 2403, M33, NGC 300, and NGC 247 represent the  $1\sigma$  uncertainties in the dust-corrected absolute J band magnitude ( $J^c$ ) due to photometric errors and reported uncertainties in Cepheid distances, added in quadrature. The Milky Way (MW) absolute magnitude assumes a Galactocenteric distance of 8.5 ± 0.5 kpc and circular speed  $\Theta_0 = 220 \pm 10$  km/s; its error bar represents the 0.23 mag uncertainty derived in section 3. The solid line is the fit to the 5 galaxy sample, excluding NGC 247 (which falls just below our magnitude limit) and the Milky Way. The dashed line is the fit to the full 7 galaxy sample.

- Fig. 2.— As figure 1, but for K band  $(2.2\mu m)$ .
- Fig. 3. As figure 1, but for L band  $(3.5\mu m)$ .





